2023

Time : 3 hours

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer from both Groups as directed.

Group – A

1. Answer **all** questions : $1 \times 12 = 12$

   (a) What is the probability of drawing two aces in succession from a pack of 52 playing cards without replacement ?

   (i) $\frac{1}{169}$

   (ii) $\frac{1}{221}$

   (iii) $\frac{1}{52}$

   (iv) None of these

(b) If x has the probability density

$$f(x) = \begin{cases} Ke^{-3x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$ then K :

(i) 1

(ii) 2

(iii) 3

(iv) 4

(c) If $F(x, y) = \begin{cases} \left(1-e^{-x}\right)\left(1-e^{-y}\right), & x>0,\ y>0 \\ 0, & \text{elsewhere} \end{cases}$

is a joint distribution function, then its joint probability density function is :

(i) e^{x+y}

(ii) e^{x-y}

(iii) e^{-x+y}

(iv) e^{-x-y}

(d) If X has varience σ^2, then var $(ax + b)$= :

(i) $a^2\sigma^2$

(ii) $a\sigma^2 + b$

(iii) $a^2\sigma^2 + b^2$

(iv) $b^2\sigma^2$

(e) If $M_x(t)$ is moment generating function, then $M(t)\ \frac{x+a}{b}$:

(i) $e^{\frac{at}{b}} M_x\left(\frac{t}{a}\right)$

(ii) $e^{at} M_x\left(\frac{t}{b}\right)$

(iii) $e^{\frac{at}{b}} M_x(t)$

(iv) $e^{\frac{at}{b}} M_x\left(\frac{t}{b}\right)$

(f) The probability given by Chebyshev's theorem is :

(i) Only upper bound

(ii) Only lower bound

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(g) Which of following distribution follows continous random variable ?

(i) Binomial distribution

(ii) Poisson distribution

(iii) Normal distribution

(iv) Hypergeometric distribution

(h) The normal regression analysis n $\frac{\sigma^{\wedge 2}}{\sigma^2}$ having chi-square distribution has _____ degree of freedom :

(i) n

(ii) n – 1

(iii) n + 1

(iv) n – 2

(i) If X has exponential distribution given by

$f(x)$ $\begin{cases} e^{-x} & , x > 0 \\ 0 & , \text{elsewhere} \end{cases}$ find probability density of random variable $y = \sqrt{x}$

(i) ye^{-y^2}

(ii) $ye^{-\sqrt{y}}$

(iii) $2ye^{-y}$

(iv) $2ye^{-y^2}$

(j) If X_r and P_x are rth and sth random variables of a random sample of size n drawn from the finite population $[C_1, C_2 \ldots\ldots C_N]$ then COV (X_r, X_s) :

(i) $\frac{-\sigma 2}{N-1}$

(ii) $\frac{\sigma 2}{N-1}$

(iii) $\frac{1}{N(N-1)}$

(iv) $\frac{\sigma 2}{N}$

(k) Which distribution is called variance ratio distribution ?

(i) t – distribution

(ii) F – distribution

(iii) Chi-Square distribution

(iv) Beta distribution

(l) The range of correlation coefficient is :

(i) [0, 1]

(ii) [–1, 1]

(iii) [– ∞, ∞]

(iv) [0, ∞]

2. Answer any **eight** questions of the following : 2×8 = 16

(a) Ant event A and ϕ are independent. Justify it.

(b) If $f(x_1, x_2, x_3) =$

$$\begin{cases} (x_1 + x_2)\, e^{-x_3}, & 0 < x_1 < 1,\ 0 < x_2 < 1,\ x_3 > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal density of x_1 and x_3.

(c) Define Random Variable with an example.

(d) If $f(x) = \begin{cases} e^x, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$, find expected value of $g(x) = e^{3x/4}$.

(e) Define disperson and write it's importance.

(f) Distinguish the difference between Poisson and Binomial distribution.

(g) Write two features of the normal distribution graph.

(h) Define law of large number.

(i) Write two importance of Least Square Method.

(j) If $\bar{X} = 16.4$, $n = 16$, $\mu = 12.0$, $S = 2.1$. Calculate the t-value.

3. Answer any **eight** questions : 3×8 = 24

(a) A die is loaded in such a way that each odd number is twice as likely to occure as each even number. What is the probability that it is a perfect square, given that it is greater than 3 ?

(b) If X and Y are independent, prove that

$$E(XY) = E(X)\, E(Y) \text{ and } \sigma_{xy} = 0.$$

(c) Prove that

$$\sigma^2 = \mu_2^1 - \mu^2$$

(d) Find μ_1^1 using the moment generating function of $f(x) = \frac{1}{8}\binom{3}{x}$, for $x = 0, 1, 2, 3$.

(e) Find covarience of :

$$f(x, y) = \begin{cases} 2, & x>0, y>0, x+y<1 \\ 0 & , \text{ elsewhere} \end{cases}$$

(f) State and prove Binomial Distribution.

(g) If X is the number of heads obtained in four tosses of a balanced coin.

Find the probability distribution of $y = \frac{1}{1+x}$.

(h) If $f(x, y) = \begin{cases} xe^{-x(1+y)}, & x>0, y>0 \\ 0 & , \text{ elsewhere} \end{cases}$. Find the regression equation of Y on X. Such the graph.

(i) If $n = 10, \{x = 20, \{x^2 = 50, \{y = 30, \{y^2 = 100, \{xy = 200$. Find correlation coefficient "r".

(j) If $f(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0 & , \text{ elsewhere} \end{cases}$. Find the probability density of $y = x^3$.

Group – B

Answer **all** questions :

4. In a certain state, 25% of all cars emit excessive amount of pollutants. If the probability is a 0.99 that a car emitting excessive amount of pollutants will fail. the state's vehicular emmission test and the probability is 0.17 that a car not emmitting the excessive amount of pollutants will neverthless fail the test. Then find the probability that a car that fails the test actually emitts excessive amount of pollutants using Bay's theorem. 7

OR

If $f(x, y) = \begin{cases} x+y, & 0<x<1, 0<y<1 \\ 0 & , \text{ elsewhere} \end{cases}$. Find the joint distribution function of these two random variables X and Y.

5. State and prove Chebyshev's theorem. 7

OR

If $f(x, y) = \begin{cases} \frac{2}{3}(x+2y), & 0<x<1, 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$

Find conditional mean and conditional variance fo X, given $Y = \frac{1}{2}$.

6. Find the moment generating function of normal distribution. 7

OR

If $f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1>0, >x_2>0 \\ 0, & \text{elsewhere} \end{cases}$ find the

(a) joint density of $y_1 = x_1 + x_2$ and $y_2 = \frac{x_1}{x_1+x_2}$

(b) Marginal density of y_2.

7. State and prove Central Limit theorem. 7

OR

Let'us consider the following data on the number of hours that 10 persons studied for a

French test and their scores on the test is given below :

Hours Studoed (x)	Test Score (y)
4	31
9	58
10	65
14	73
4	37
7	44
12	60
22	91
1	21
17	84

Find the equation of the least squares line that approximate the regression of

the test scores on the number of hours studied. Also, predict the average test scores of a person who studied 14 hours for the test.